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ASYMMETRIC ADJUSTMENT AND NONLINEAR DYNAMICS IN REAL EXCHANGE RATES

HYGINUS LEON[a,*,†] and SERINEH NAJARIAN[b]
[a] *International Monetary Fund, Washington, DC, USA*
[b] *Hertford College, Oxford, UK*

ABSTRACT

This paper examines whether deviations from PPP are stationary in the presence of nonlinearity, and whether the adjustment towards PPP is symmetric from above and below. Using alternative nonlinear models, our results support mean reversion and asymmetric adjustment dynamics. We find differences in magnitudes, frequencies and durations of the deviations of exchange rates from fixed and time-varying thresholds, both between over-appreciations and over-depreciations and between developed and developing countries. In particular, the average cumulative sum of deviations during periods when exchange rates are below forecasts is twice that during periods of over-appreciation and larger for developing than advanced countries. Copyright © 2005 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Purchasing power parity (PPP) states that national price levels should be equal when expressed in a common currency. Therefore, variations in the real exchange rate (RER), defined as the nominal exchange rate adjusted for relative national price levels, represent deviations from PPP. While an exact PPP relationship is not expected to hold at every period, researchers have been concerned about the almost universal finding that deviations from PPP appear to persist for very long periods (that is, have unit roots). Sarno and Taylor (2002) list three reasons why we should care if the real exchange rate has a unit root. First, the degree of persistence can be used to infer the principal impulses driving real exchange rate movements, high persistence indicating principally supply side shocks. Second, nonstationarity questions a large part of open economy macroeconomic theory that assumes PPP. Third, economic policies based on estimates of PPP exchange rates may be flawed if the real exchange rate contains a unit root. Research on PPP has therefore focused on the credibility of the unit root finding and on why deviations from PPP exist.

One explanation of the unit root finding relates to the low power of unit root tests. Consequently, a number of researchers have sought to increase the power of unit root tests by increasing the span of the data (Lothian and Taylor, 1996; Cheung and Lai, 1998), and by using panel unit root tests (Frankel and Rose, 1996; Taylor and Sarno, 1998). Another explanation, which we examine in this paper, is that standard unit root tests are likely to be biased and have low power in rejecting the null of a unit root because real exchange rates follow a nonlinear adjustment process (Yilmaz, 2001; Bergman and Hansson, 2000; DeGrauwe and Vansteenkiste, 2001; Kilian and Taylor, 2002; Michael *et al*., 1997; Taylor, 2001).

*Correspondence to: Hyginus Leon, International Monetary Fund, 700 19th street, NW, Washington, DC 20431, USA.
†E-mail: hleon@imf.org

1.1. *Nonlinear adjustment and asymmetry*

Nonlinear exchange rate adjustment may arise from transaction costs in international arbitrage (Sercu *et al*., 1995; Obstfeld and Taylor, 1997; Coleman, 1995; O'Connell and Wei, 2002).[1] Deviations from PPP are assumed not corrected if they are small relative to the costs of trading.[2] Proportional or 'iceberg' costs create a band (thresholds) for the real rate, within which the marginal cost of arbitrage exceeds the marginal benefit. Dixit (1989) and Krugman (1989) argue that thresholds may also arise because of sunk costs of international arbitrage and the tendency for traders to wait for sufficiently large arbitrage opportunities before entering the market. Another explanation is government intervention, which is more likely the farther the exchange rate is from its desired or target rate (Dutta and Leon, 2002).[3] Governments care about large and persistent deviations because real exchange rates are likely to affect net exports, as well as the cost of servicing debt denominated in foreign currency. In fact, Calvo *et al*. (1995) concluded that the RER is perhaps the most popular real target in developing countries. Similarly, Lundbergh and Teräsvirta (2003) propose a smooth transition autoregressive target zone model to characterize the dynamic behaviour of an exchange rate fluctuating within a target zone, where the degree of change depends nonlinearly on the distance between the value of the process and the central parity of the target zone. Almekinders and Eijffinger (1996) find that the US and German central banks 'leaned-against-the-wind' selectively in the post-Louvre period and tried to counteract appreciations of their currency more strongly than depreciations, suggesting asymmetry in intervention behaviour.

These models suggest that the exchange rate can be modelled as a regime-switching process, with a band of inaction. Thus, the exchange rate will at least revert to a range. Two issues arise: the choice of the switching function governing the regime change and the symmetry of rates of reversion on either side of the band of inaction. In some models the jump to mean or range reversion is sudden (Obstfeld and Taylor, 1997), while in others it is smooth (Michael *et al*., 1997). Dumas (1992) and Teräsvirta (1994) argue that time aggregation and nonsynchronous trading favour smooth transition between regimes. It can also be argued that the averaging implicit in the compilation of the real exchange rate index would suggest a smooth rather than discontinuous adjustment process, given that the underlying goods traded have different arbitrage costs. For the second issue, the accepted view is that the transactions–cost model requires symmetry of thresholds and adjustment parameters (Lo and Zivot, 2001). For example, Michael *et al*. (1994) argue that because adjustment to PPP deviations must be the same for positive and negative deviations from equilibrium, it is appropriate to specify a symmetric threshold autoregressive (TAR) model with the same autoregressive parameters in the outer regimes. Similarly, Sarno *et al*. (2001) propose a nonlinear model that implies random behaviour near equilibrium but mean-reverting behaviour for large departures from fundamentals. They estimate an exponential threshold autoregressive (ESTAR) model that implies symmetric adjustment of the exchange rate above and below equilibrium.[4] On the other hand, Dutta and Leon (2002) argue that countries may choose to defend depreciations more or less vigorously than appreciations, thereby generating asymmetric adjustment behaviour.

We address these issues by estimating and evaluating three classes of regime switching models for a range of advanced and developing economies.[5] The first model is a time-varying threshold autoregressive model (TVTAR), which allows asymmetrical adjustment when real exchange rates deviate from forecasts. The estimated model allows us to calculate the magnitudes, frequencies and durations of these deviations from forecasts, both for depreciations and appreciations. The second specification is an adaptation of Silverstovs' (2000) bi-parameter smooth transition regression (BSTR), which allows for asymmetric adjustment between the middle and outer regimes. The third specification is a Markov switching model (MSM), where the change in the regimes in exchange rate dynamics is governed by an unobservable Markov chain. Thus, our design compares smooth versus sudden switching in regimes, includes fixed and time-varying thresholds, and allows for asymmetry in adjustment.

1.2. *Issues in testing for unit roots*

If the true model is nonlinear, then estimates from a linear model will average the potentially reverting data outside of the band with the nonstationary nonreverting data within the band, leading to biases, especially if bias due to nonlinearity interacts with bias due to temporal aggregation (Taylor, 2001). Therefore, the effect of nonlinearity needs to be considered in tests for nonstationarity. Pippenger and Goering (1993, 2000) argue that the presence of threshold nonlinearities reduces the power of standard unit root and cointegration tests; Michael *et al*. (1997) argue that cointegration or unit root tests may be biased when the linear alternative neglects nonlinearity of the smooth transition autoregressive (STAR) type. Nelson *et al*. (2001) show that standard augmented Dickey–Fuller (ADF) tests have low power against stable but occasionally integrated alternatives. In fact, these nonlinear models may be globally stationary even if they have a unit root in the middle regime.

Testing for unit roots when the DGP is nonlinear poses two problems. First, which nonlinear model should be used? Most researchers consider one process and few comparisons exist (see Carrasco, 2002; Taylor and van Dijk, 2002). Yet, the failure to confirm a regime shift may be due to misspecification of the alternative. Second, how should we test for nonstationarity in the presence of nonlinearity? We address these issues by estimating alternative nonlinear specifications and employing recent developments in the joint analysis of nonstationarity and nonlinearity, proposed by Balke and Fomby (1997) in the context of threshold cointegration and subsequently developed by Berben and van Dijk (1999), Caner and Hansen (2001), Kapetanios and Shin (2002) and Lo and Zivot (2001) in the context of TAR models, and by Kapetanios *et al*. (2003) when the alternative is a stationary ESTAR process.

1.3. Summary of results

Our research contributes to the literature in three related ways. First, we introduce the TVTAR and provide evidence on nonstationarity in the presence of nonlinearity. For the TVTAR models, we follow Caner and Hansen (2001), who allow for both effects simultaneously, in computing Wald tests for unit roots (nonstationarity) when the threshold nonlinearity is either present or absent.[6] Second, we focus on potential asymmetries in the short-run dynamics of real exchange rates by allowing the parameters of the models to be estimated unrestrictedly. In particular, we estimate BSTR models that allow different adjustment speeds from the lower-to-middle and middle-to-upper regimes, providing direct evidence on asymmetrical adjustment. Third, we implement tests that allow comparisons of alternative specifications. We follow Breunig *et al*. (2002) (BNP) who develop tests to compare the implied densities of the estimated models with that of the data. We complement the BNP tests with Hamilton's (2001) flexible parametric nonlinearity test and Li's (1996) test of density equivalence.[7]

We estimate the models for 26 countries, using monthly data on real effective exchange rates. Our sample includes all G7 countries, a selection of advanced countries, and some emerging market countries from Asia and Latin America.[8] Our results provide support for both *stationary* regime-switching processes and asymmetric adjustment. For the threshold models, the Wald tests show that the unrestricted TVTAR outperforms both the linear specifications (stationary as well as nonstationary) and the identified threshold nonstationary model (unit root with threshold effects). We find support in some developing countries for the threshold model with a unit root in the corridor regime. For the smooth transition models, we find reversion to the mean in almost every case when the nonlinear component is included. As regards *asymmetry,* we calculate the speed of response to deviations from forecasts and duration of time spent outside threshold bands to gauge the potential impact of real exchange rate misalignments. For the TAR models, we find that while advanced countries *respond* faster than developing countries to over-appreciations and over-depreciations, Asian and G7 (other advanced and Western Hemisphere) countries in our sample respond more strongly to developments relating to over-appreciations (over-depreciations). We find asymmetric speeds of adjustment between regimes in the smooth transition models in more than half of the countries. In general, *durations* are longer for over-appreciations. For the threshold model, durations are longer for over-depreciations in the G7 and Asian countries, but for over-appreciations in the other advanced countries and countries in the Western Hemisphere (WH). The *excess deviation* measure of these over-depreciations is uniformly about twice that for over-appreciations and larger for developing countries than for advanced countries. We *evaluate* all the models estimated for their ability to replicate five characteristics of the densities of the data. We find that the nonlinear specifications better explain the first two moments and the asymmetry and persistence characteristics to a lesser extent, but do less well, especially for developing countries, in replicating the observed interquartile range. In general, the BSTR specification, which captures best the characteristics of interest, adequately characterizes the nonlinearity in the observed data and provides a realistic insight into the short-run dynamics.

Some potential implications of our results relate to the effects of real exchange rate misalignment. Countries with longer durations of misalignment, larger deviations from threshold bands, or higher excess deviations could have a higher probability of experiencing hysteresis effects. These probabilities appear higher for over-depreciations than for over-appreciations and more so for developing countries than for advanced economies. Consequently, an argument can be made for interventionist policies aimed at reducing variability and length of duration of misalignments outside a desired range.

The rest of this paper is organized as follows. Section 2 discusses the nonlinear frameworks used in estimating the real exchange rate dynamics. In Section 3 we present the results. A brief summary follows in Section 4.

2. NONLINEAR FRAMEWORKS

Nonlinear modelling of economic variables assumes that different states of the world or regimes exist and that the dynamic behaviour of economic variables depends on the regime occurring at a point in time. Therefore certain properties of the time series, such as means and autocorrelations, vary with each regime. We consider nonlinear models that are characterized as piecewise linear processes, such that the process is linear in each regime. Each model is distinguished by a different stochastic process governing the change of regime. Our models are intentionally eclectic and nonnested to provide a measure of robustness to the results. Our generic functional form is:

$$y = \tilde{\pi}_1' \tilde{Z}_t + \tilde{\pi}_2' \tilde{Z}_t \Phi(v_t; \psi) + \xi_t$$

where y_t is the dependent variable of interest, $\tilde{Z}_t$ is a vector of lagged dependent variables, $\tilde{\pi}_j$ are the parameter vectors, Φ is the regime-switching function, v_t is the transition variable, ψ is the threshold vector and $\xi_t \sim$ i.i.d. $(0, \sigma^2)$. Thus, each model reduces to a linear process under the null hypothesis $\tilde{\pi}_2 = 0$. We consider two classes of regime-switching models. The first class assumes that the regimes are determined by an observable variable. We examine a threshold model, with a discrete jump at a threshold value, and a smooth transition model, with a continuous function determining the weight assigned to the regimes. In both models, the switching function is dependent on the value of the transition variable relative to the threshold value. In the second class, the regimes are not observed but are inferred from an unobservable stochastic process. We examine the Markov-switching model, with changes driven by an unobservable exogenous Markov chain, S_t.

In all three models, testing is problematic because of nuisance parameters in the transition function, which are identified only under the alternative (Davies, 1987; Hansen, 1997). In the threshold and smooth transition models, the nuisance parameters are the parameters of the transition function (values of the thresholds and delay factor of the transition variable), while in the Markov-switching model, the nuisance parameters are the transition probabilities.

2.1. *Threshold autoregressions (TAR)*

In the TAR model, introduced and popularized by Tong (1978) and Tong and Lim (1980), the parameters of the process generating the data depend on the value of the regime-switching variable. The series can then be categorized into states consistent with the threshold variable reaching the threshold values separating the regimes. In the context of real exchange rates, the TAR model allows for a band within which no adjustment to the deviations from PPP takes place. This implies that within the band, deviations from PPP may exhibit unit root behaviour, but the adjustment process is reverting or stationary in the outer bands. Because the bands of inaction may vary over time, due to changes in relative transactions costs, other market frictions and/or policy intervention, Leon and Najarian (2003) introduce and estimate the following time-varying TAR (TVTAR):

$$\begin{aligned}\Delta y_t &= \theta'_L x_{t-1} I_{t,L} + \theta'_H x_{t-1} I_{t,H} + \theta'_C x_{t-1} + \varepsilon_t \\ x_{t-1} &= (1, y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-k}), \quad \theta'_R = (\beta_{0R}, \rho_R, \beta_{1R}, \ldots, \beta_{kR}), \quad R = L, C, H\end{aligned} \tag{1}$$

and

$$I_{t,L} = \begin{cases} 1 & \text{if } z_t < 0 \wedge |z_t| > |P_{t-1,L}(z_t)| \\ 0 & \text{otherwise} \end{cases}$$

$$I_{t,H} = \begin{cases} 1 & \text{if } z_t > 0 \wedge |z_t| > |P_{t-1,H}(z_t)| \\ 0 & \text{otherwise} \end{cases}$$

For $z_t = \Delta y_{t-1}$,

$$P_{t-1R}(z_t) = \alpha_{t-1,R}(z_{t-1}) + (1 - \alpha_{t-1,R}) P_{t-2,R}(z_{t-1})$$

$$\alpha_{t-1,R} = \left| \frac{S_{t-1,R}}{A_{t-1,R}} \right|$$

with

$$S_{t-1,R} = \delta_R dev_{t-1,R} + (1 - \delta_R) S_{t-2,R}$$

$$A_{t-1,R} = \delta_R |dev_{t-1,R}| + (1 - \delta_R) A_{t-2,R}$$

and

$$dev_{t-1,R} = z_{t-1} - P_{t-2,R}(z_{t-1})$$

$P_{t-1}(z_t)$ is the expected forecast value of the transition variable, based on exponential smoothing with adaptive response (time-varying) weights for the exponential rate of decay. Thus, the three-regime TVTAR divides the regression according to whether the absolute value of the percentage change in the real exchange rate exceeds the upper and lower forecast bounds, $P_{t-1,R}(z_t)$. The corridor regime occurs when the change in the real exchange rate during one month does not appreciate by more than the upper forecast bound, $P_{t-1,H}(z_t)$, or depreciate by more than the lower forecast bound, $P_{t-1,L}(z_t)$. The transition variable $z_t = \Delta y_{t-d}$ is assumed to be known, stationary and have a continuous distribution; however, the delay factor d, the lag length k and the threshold values are unknown. Each δ_L, δ_H depends on a functional of the sample. $I(A)$ denotes the indicator function for the event A, such that $I(A) = 1$ if A is true and $I(A) = 0$ otherwise. In interpreting the coefficients, R is an index for the alternative regimes, ρ_R are the slope coefficients on y_{t-1}, β_{0R} are the slope coefficients on the deterministic components and β_{iR} are the slope coefficients on the $(\Delta y_{t-1}, \ldots, \Delta y_{t-k})$ in the alternative regimes. The model can be nonstationary within one or more regimes, though the alternation between regimes can make it overall stationary.

Unit root tests. Following Caner and Hansen (2001), Leon and Najarian (2002) compute the following Wald statistics for distinguishing between nonlinearity (threshold effects) and possible nonstationarity (unit roots) in real exchange rate series:[9]

Wald 1: linear stationary-ergodic AR versus unrestricted TAR

$$H_0 : \theta_L = \theta_H = 0, \rho_C < 0$$
$$H_A : \theta_L \neq 0, \theta_H \neq 0$$

Wald 2: Hansen's unidentified threshold scenario

$H_0: \theta_L = \theta_H = 0, \rho_C = 0$

H_A: unrestricted three-regime TAR

Wald 3: Hansen's identified threshold

$H_0: \theta_L \neq 0,\ \ \theta_H = 0,\ \ \rho_L = \rho_H = \rho_C = 0$

$H_A: \theta_L \neq 0,\ \ \theta_H \neq 0,\ \ \rho_L < 0,\ \ \rho_H < 0,\ \ \rho_C < 0$ (unrestricted three-regime TAR)

Wald 4: unit root in corridor regime, partial unit root

$H_0: \theta_L \neq 0,\ \ \theta_H \neq 0,\ \ \rho_L = \rho_H = \rho_C = 0$

H_A: unrestricted three-regime TAR

The test is an *F*-statistic calculated as the ratio of residual variance of the linear model (null) to that of the TAR model (alternative); however, the *F*-statistic does not have the standard χ^2 (chi-square) asymptotic distribution. Given the dependence of the critical values on the particular null and alternative, as well as the presence of nuisance (unidentified under the null) parameters, we calculate the critical values for the test statistics using bootstrap approximations to the asymptotic distributions of the Wald statistics.[10] The unidentified threshold scenario, which performed better in Caner and Hansen's (2001) Monte Carlo tests, makes use of the constrained bootstrap method,[11] and the identified threshold bootstrap is conducted through a simulation from a unit root TAR. The Wald 1 is a test for the existence of a threshold; Wald 2 tests for a unit root when there is no threshold effect; Wald 3 tests for a unit root in the presence of threshold effects; and Wald 4 tests for a (partial) unit root only in the corridor regime.

2.2. *Smooth transition regressions (STR)*

In contrast to the TAR model, where the switch between regimes occurs abruptly at a specific value of the threshold variable, smooth transition regression models allow a more gradual transition between regimes. STR models, introduced by Chan and Tong (1986) and popularized by Granger and Teräsvirta (1993), are a more general class of state-dependent nonlinear time series models capable of accounting for deterministic changes in parameters over time, in conjunction with regime-switching behaviour (see survey in van Dijk *et al*., 2002). The STR model can be viewed as a weighted average of two linear models, with weights determined by the value of a transition function, typically defined as either a logistic or an exponential function.[12]

The STR model of order *r* is:

$$\Delta y_t = \theta_1' x_{t-1} + \theta_2' x_{t-1} F(z_t^d; \gamma, c) + \mu_1 \tag{2}$$

where x_{t-1}, defined as in equation (1), is a vector of exogenous variables; z_t^d is the transition variable and may include a linear combination of several variables; F is the transition function determining the weights of the regimes and is bounded between 0 and 1; γ measures the speed of transition from one regime to the next; and c is the location variable (threshold) for the transition function. As γ becomes very large, the change of $F(z_t^d; \gamma, c)$ from 0 to 1 becomes almost instantaneous at $z_t^d = c$, and the transition function approaches the indicator function $I[z_t^d > c]$. The conventional STAR model is a special case of the smooth transition model when $z_t^d = \Delta y_{t-d}$.

A natural counterpart to the multiple regime TAR model is the multiple regime smooth transition autoregressive (MSTAR) model, which has multiple transition functions, each with its own location and slope parameters. Silverstovs (2000) argues that the greater flexibility of the MSTAR model may also be a drawback in the case of a three-regime model with two identical outer regimes and with asymmetric speed of transition between regimes. He proposes the bi-parameter smooth transition regression (BSTR) model, with the following transition function:

$$F_t(\gamma_1, c_1, \gamma_2, c_2; z_t^d) = \frac{\exp[-\gamma_1(z_t^d - c_1)] + \exp[\gamma_2(z_t^d - c_2)]}{1 + \exp[-\gamma_1(z_t^d - c_1)] + \exp[\gamma_2(z_t^d - c_2)]}, \quad \gamma_1, \gamma_2 > 0; \ \ c_1 < c_2 \tag{3}$$

where γ_1 and γ_2 determine the speed of transition at their corresponding transition locations. In particular, the slopes of the transition functions at the two threshold parameters are different, thus allowing the transition speed from the lower-outer to middle regime and from the middle to higher-outer regime to be asymmetric. With four parameters, the BSTR(p) offers a large variety of shapes, with the magnitude of each slope parameter determining the steepness of the slope of the transition function.[13] Smooth transition models are arguably more appropriate in modelling foreign exchange markets than threshold autoregressive or Markov regime-switching models because of the large number of investors, different investment horizons and varying learning speeds, which suggest smooth rather than discrete adjustment.

Estimation. After determining the transition function and the threshold variable, the parameters of an STR model can be estimated by nonlinear least squares (NLS). For $y_t = F(x_t;\theta) + \varepsilon_t$, the NLS estimator is given by $\hat{\theta} = \arg\min_\theta \sum_{t=1}^{T} (y_t - F(x_t;\theta))^2 = \arg\min_\theta \sum_{t=1}^{T} \varepsilon_t^2$. If ε_t is normal, NLS is equivalent to maximum likelihood (MLE). Otherwise, NLS can be interpreted as a quasi-maximum likelihood estimator (QMLE). Pötscher and Prucha (1997) demonstrate that NLS is consistent and asymptotically normal under appropriate regularity conditions.

2.3. *Markov-switching models (MSM)*

In Markov-switching models, the parameters of the process generating the dependent variable depend on the unobservable regime variable, S_t, which indicates the probability of being in a particular state of the world.[14] The process generating a change in regime depends on an exogenous unobservable Markov chain. Here we model real exchange rate appreciations and depreciations as switching regimes of the stochastic process underlying the data generating process. Thus appreciations and depreciations are associated with different conditional distributions of the change in the real exchange rate. The parameters of each regime are estimated unrestrictedly.

We consider

$$\begin{aligned}\Delta y_t &= \sum_{R=1}^{m} E[\Delta y_t|S_t = R; \Delta\tilde{y}_{t-1}]\Pr(S_t = R|\Delta\tilde{y}_{t-1}) \\ &= \sum_{R=1}^{m}\left(\alpha_R + \rho_R\, y_{t-1} + \sum_{i=1}^{k}\beta_{iR}\Delta y_{t-i}\right)\Pr(S_t = R|\Delta\tilde{y}_{t-1}) + \sigma v_t, \quad v_t \sim N(0,1)\end{aligned} \tag{4}$$

where $\Delta\tilde{y}_{t-1} = (\Delta y_{t-1}, \Delta y_{t-2}, \ldots, \Delta y_{t-(t-1)})$, and S_t is a three-state Markov chain with unknown transition probabilities P_{ij}, given by $P_{ij} = \Pr(S_t = j|S_{t-1} = i)$. Thus, the conditional density is weighted by the predicted probability of being in a specific regime at time t, given the information set. The sequence of predicted probabilities, which indicate the likelihood of the variable being in a particular state in each time period, is:

$$\Pr(S_t|\Delta\tilde{y}_{t-1}) = \frac{P^T[\Pr(\Delta y_{t-1}|\Delta\tilde{y}_{t-2}, S_{t-1}) \otimes \Pr(S_{t-1}|\Delta\tilde{y}_{t-2})]}{\{\Pr(\Delta y_{t-1}|\Delta\tilde{y}_{t-2}, S_{t-1})^T \Pr(S_{t-1}|\Delta\tilde{y}_{t-2})\}}$$

where $\otimes$ denotes element-wise matrix multiplication. To illustrate, we consider a simple two-state model where states (regimes) alternate between zero and unity. Then:

$$\Delta y_t = \theta_1' x_{t-1}(1 - S_t) + \theta_2' x_{t-1} S_t + \varepsilon_t \tag{5}$$

The null hypothesis of linearity can generally be formulated in terms of restrictions on θ_1 or θ_2, leaving the transition probabilities unidentified. This well-documented identification problem poses a challenge for conventional specification and evaluation tests.

The parameters of the model are estimated by maximum likelihood, with normality assumed to ensure consistency. Because S_t is not observed, inference about the states is carried out using an expectation maximization (EM) algorithm, with smoothed probabilities of the unobserved states replacing the conditional regime probabilities in the likelihood function.[15] Critical values for the test statistics are generated by simulation methods.

2.4. *Model evaluation*

Despite the recent proliferation in the use of nonlinear models, the relative merits of alternative classes of models still remain a nontrivial problem because alternative specifications are not nested and the use of standard asymptotic theory is often highly questionable. Most specification tests for nonlinear models tend to be based on time series analysis of standardized residuals. Breunig *et al*. (2002) (BNP) argue that because formal procedures such as likelihood ratio tests of hypotheses may be difficult to interpret in nonlinear models, given their sensitivity to particular observations, it is necessary to complement these procedures with informal methods of evaluation. For example, if the act of simulating a model demonstrates that there is a fundamental flaw with it, this raises doubts about the validity of the maximum likelihood theory used in constructing a formal test (see Breunig and Pagan, 2001; Pagan, 2001). BNP (2002) develop tests based on simulations of models that allow the discovery of population characteristics that can be compared with the corresponding sample equivalents. These tests allow us to compare the performance of the competing nonlinear models without *a priori* assumptions that either model is the true data generation process (DGP). This is particularly important because most times the researcher does not know which model may have generated the hypothesized shift in regime.

If our focus is the DGP, it is natural to focus on the density describing the variable of interest. Because the density is generally unknown, we have to estimate it, preferably with an estimator that does not already assume that the null hypothesis is correct. One way of doing this is to use a nonparametric estimator—that is free from all parametric assumptions regarding the moments of the distribution—which will converge to the true density whether or not the parametric model is correctly specified. We can compare this density with that implied by the estimated model. Clearly, the density implied by the estimated model will converge to the true density only if the model is correctly specified. A measure of the distance between the two density estimates provides a natural statistic to test the null hypothesis of correct parametric specification. Ait-Sahalia (1996) uses this notion to compare a nonparametric density estimate with a parametric density estimate from the estimated parametric model. In contrast, we report results for a test of closeness between two unknown density functions, due to Li (1996), which compares an empirical density (nonparametric kernel) to a nonparametric density based on simulated data from the estimated models.

In practice, researchers tend to focus on some characteristics of the density, depending on the objectives of the modelling exercise. For example, these may include the conditional mean (if the objective is prediction of a point estimate), volatility (if our interest is uncertainty), skewness (if interest is in the relative balance of upside and downside risk), kurtosis (if our interest is in the impact of very large changes) and asymmetry (if we are interested in distinguishing potentially different magnitudinal effects). So, suppose the analyst (policy maker) is interested in some functions of data, $\hat{g}(y)$. Let $g(\hat{\theta})$ be the corresponding implied population characteristic, obtained from simulated data based on the estimated model. Label the difference between these two measures as $d = \hat{g}(y) - g(\hat{\theta})$. Then, we can think of these tests as comparing a consistent estimator of $g(y)$ to an efficient estimator, $g(\hat{\theta})$, if the model is valid, enabling us to formulate the variance of d as $\mathrm{var}(d) = \mathrm{var}(\hat{g}(y) - \mathrm{var}(g(\hat{\theta}))$ (see Hausman, 1978). Although the variance of $\hat{g}(y)$ is simply derived from the observed series, the analytical expression for $\mathrm{var}(g(\hat{\theta}))$ may be difficult to obtain for complicated nonlinear specifications. Because the test statistic $T^* = \hat{d}'[\mathrm{var}(\hat{g}(y) - \mathrm{var}(g(\hat{\theta}))]^{-1}\hat{d} > T = \hat{d}'[\mathrm{var}(\hat{g}(y))]^{-1}\hat{d}$, Pagan (2002) suggests using the conservative test T. A rejection based on T (compared to χ^2 (1)) would imply an even stronger rejection than if based on T^*. A robust estimator of $\mathrm{var}(\hat{g}(y))$, compatible with many alternative models, can be obtained using the Newey–West (1987) covariance matrix.

3. ESTIMATION AND RESULTS

We examine real effective exchange rates for 26 countries, 13 of which are industrial countries.[16,17] All data are taken from the *International Financial Statistics* (*IFS*) database of the International Monetary Fund (IMF). The real effective exchange rate (REER), based on consumer prices, measures movements in the nominal exchange rate adjusted for differentials between the domestic price index and trade-weighted foreign price indices. The IMF's CPI-based REER indicator (year $1995 = 100$) of country i is:

$$e_t = \prod_{j \neq i} \left(\frac{P_i R_i}{P_j R_j} \right)^{W_{ij}}$$

where j is an index of country i's trade partners, W_{ij} is the competitiveness weight put by country i on country j, P_i and P_j are consumer price indices in countries i and j, and R_i and R_j represent the nominal exchange rates of countries i and j's currencies in US dollars. An increase (appreciation) in a country's index indicates a decline in international competitiveness.

A preliminary evaluation of the data shows that real exchange rates in the developing countries in our sample are more volatile (have higher standard deviations) than those of the advanced countries. Their distributions are also more skewed. Nonnormality is common across all regions.[18] We calculate both the Dickey–Fuller (ADF) and Ng and Perron (2001) unit root tests, given the significant moving average coefficients found in estimated ARMA (1,1) models. We find that, except for Brazil and Costa Rica (using ADF), we cannot reject the unit root hypothesis (see Table 1). As indicated earlier, these conventional tests, which do not account for nonlinearity, may be misleading; however, our initial unit root results are

Table 1. Descriptive statistics

	Moments and unit root tests											
	Mean	SD	Skew	KT	J-B	MZa	MZt	MSB	MPT	ADF	MA	MA_t
Advanced *G7*												
Canada	4.49	0.11	−0.12	1.60	22.07	−0.87	−0.41	0.47	15.06	−1.09	0.26	4.18
France	4.59	0.04	−0.06	3.31	1.23	−1.79	−0.61	0.34	9.75	−2.25	0.31	5.09
Germany	4.61	0.05	0.29	2.58	5.66	−6.83	−1.82	0.27	3.70	−2.23	0.29	4.69
Italy	4.46	0.08	0.51	2.58	13.17	−2.98	−1.22	0.41	8.20	−1.83	0.52	9.41
Japan	4.67	0.20	−0.50	2.14	18.95	−2.06	−0.93	0.45	11.07	−1.84	0.39	6.49
United Kingdom	4.59	0.09	−0.10	1.83	15.46	−2.05	−1.01	0.49	11.96	−1.73	0.36	5.95
United States	4.70	0.12	0.64	2.36	22.32	−3.25	−1.13	0.35	7.41	−1.95	0.39	6.61
Other												
Australia	4.55	0.13	0.31	1.98	15.69	−0.79	−0.38	0.48	15.80	−1.73	0.37	6.22
Belgium	4.59	0.05	0.44	2.93	8.56	−5.77	−1.60	0.28	4.55	−2.82	0.30	4.80
Israel	4.62	0.07	0.43	2.34	13.10	−3.83	−1.17	0.30	6.56	−2.12	0.27	4.25
Korea	4.53	0.12	−1.07	4.25	67.05	−5.66	−1.59	0.28	4.62	−2.30	0.60	11.42
New Zealand	4.55	0.09	0.09	2.52	2.82	−7.61	−1.86	0.24	3.56	−1.99	0.43	7.40
Spain	4.45	0.09	0.26	2.89	3.09	−4.08	−1.43	0.35	6.01	−1.72	0.37	6.14
Developing *Asia*												
India	4.65	0.38	0.31	1.39	32.79	0.60	0.96	1.59	152.66	−1.62	0.13	2.05
Indonesia	4.72	0.44	0.02	2.65	1.38	0.10	0.07	0.70	31.95	−1.37	0.16	2.55
Malaysia	4.71	0.18	0.34	2.08	14.52	−0.92	−0.49	0.54	17.55	−1.28	0.25	3.94
Philippines	4.79	0.16	0.56	2.22	20.53	−0.83	−0.44	0.53	17.51	−1.93	0.22	3.46
Thailand	4.65	0.16	0.13	2.62	2.29	−0.04	−0.02	0.68	29.09	−1.43	0.27	4.21
WH												
Argentina	4.87	0.37	−0.63	2.44	20.67	−2.62	−1.14	0.44	9.37	−1.96	0.03	0.41
Brazil	4.33	0.19	−0.03	1.78	16.25	−10.4	−2.17	0.21	2.78	−2.30	0.29	4.53
Chile	4.88	0.25	1.09	3.24	52.79	0.12	0.12	0.97	55.03	−2.47	0.11	1.66
Colombia	4.98	0.25	0.46	2.04	19.48	−0.33	−0.28	0.83	37.88	−1.97	0.45	7.80
Costa Rica	4.65	0.13	0.64	9.45	474.7	−2.21	−0.82	0.37	9.50	−4.61	0.46	7.72
Mexico	4.68	0.20	−0.30	2.45	7.14	−4.37	−1.47	0.34	5.62	−2.67	0.30	4.86
Paraguay	4.89	0.24	0.76	2.44	28.50	0.77	0.98	1.28	104.93	−1.84	0.12	1.87
Uruguay	4.96	0.23	−0.05	1.50	24.85	−2.60	−1.14	0.44	9.44	−1.78	0.04	0.65

Note: SD is the standard deviation, KT is kurtosis and J-B the Jarque–Bera normality test. The Ng and Perron (2001) tests reported are modified forms of the Phillips and Perron Za and Zt statistics, the Bhargava (1986) R1 statistic, and the Elliot *et al.* (1997) point optimal statistic. The 5% critical values are −8.10 for Mza, −1.98 for MZt, 0.23 for MSB, 3.17 for MPT and −2.87 for ADF.

consistent with the existing literature. In what follows we estimate nonlinear models and re-evaluate the evidence for the unit root hypothesis.

3.1. TAR estimates

We estimate equation (1) using sequential least squares (Hansen, 1997), for the period 1981:03 to 2001:12, with Ox Professional 3.0. Our δ_R are initialized through a grid search over [0,1] in steps of 0.1 increments, determining the α_R, the threshold sequences and the indicator variables (I_L, I_H). We use the lagged difference of the exchange rate as the transition variable and set the delay parameter to unity.[19] Our choice of $z_t = \Delta y_{t-1}$ is stationary whether y_t is $I(1)$ or $I(0)$. We also initialize $S_{t-2,R} = 0$, $A_{t-2,R} = 0$ and $F_{t-2,R} = \Delta y_{t-2}$. For each triple (δ_L, δ_H, k), consisting of the lower and upper thresholds and lag k on Δy_{t-k}, we estimate by OLS:[20]

$$\Delta y_t = \hat{\theta}'_L(\delta_L, \delta_H, k)x_{t-1}I_{t,L} + \hat{\theta}'_H(\delta_L, \delta_H, k)x_{t-1}I_{t,H} + \theta'_C(\delta_L, \delta_H, k)x_{t-1} + \varepsilon_t(\delta_L, \delta_H, k)$$

Let $\sigma^2(\delta_L, \delta_H, k) = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_t(\delta_L, \delta_H, k)^2$ be the OLS estimate of σ^2 for fixed δ_L, δ_H, k. Then the least squares estimate of the threshold values is found by minimizing $\sigma^2(\delta_L, \delta_H, k)$:

$$(\hat{\delta}_L, \hat{\delta}_H, \hat{k}) = \arg\min_{(\delta_L,\delta_H,k)\in\Lambda_L\Lambda_H\Lambda_K}\hat{\sigma}^2(\delta_L, \delta_H, k)$$

The parameters of the model can be estimated consistently as long as the true threshold values lie in the interior of the grid space and each regime has sufficient data points to produce reliable estimates of the autoregressive parameters. The least squares estimates of the other parameters and residuals are found by substitution of the point estimates $(\hat{\delta}_L, \hat{\delta}_H, \hat{k})$.

Empirical characteristics. We investigate estimated lag lengths, speed of response to deviations from forecasts, time spent outside threshold bounds, and a measure of deviations between actual changes and forecast thresholds during periods outside of thresholds. We present results for groupings of advanced and developing countries. Summaries of the characteristics of the threshold bands and estimates of duration are shown in Tables 2 and 3 and described below.[21]

Lag length: On average, the specifications for developing countries are characterized by longer lags of exchange rate changes. The average lag for advanced countries is 5 compared to 6.8 for developing countries. For countries in the Western Hemisphere (WH), the average lag is as high as 7.3. This suggests a more complex structure for short-term interaction between nominal exchange rates and relative prices; it also highlights the importance of correct lag length in tests of unit roots because omission of short-run dynamics could affect tests based on the long-term impact matrix (see the Π matrix in the Johansen test).

Response: The adaptive response weight parameters α_L and α_H show the quickness of response to relatively recent exchange rate variations. Advanced countries respond faster than developing countries to both over-depreciations (0.56 vs. 0.50) and over-appreciations (0.53 vs. 0.47), implying narrower and

Table 2. Characteristics of threshold bands

	δ_L	δ_H	α_L	α_H	κ^*	$\%L$	$\%H$	$\%Cor$
Advanced	0.38	0.52	0.56	0.53	5.00	0.27	0.28	0.45
G7	0.34	0.54	0.51	0.55	5.14	0.29	0.27	0.45
Other	0.43	0.48	0.62	0.51	4.83	0.26	0.29	0.46
Developing	0.31	0.37	0.50	0.47	6.77	0.26	0.30	0.45
Asia	0.42	0.44	0.45	0.63	6.00	0.28	0.26	0.47
WH	0.24	0.33	0.53	0.38	7.25	0.24	0.33	0.43
Overall	0.35	0.44	0.53	0.50	5.88	0.26	0.29	0.45

Note: Let subscript R depict the alternative regimes, with L corresponding to over-depreciation, H to over-appreciation and Cor to the corridor. The columns report the parameters from the forecast measure that characterizes the time-varying bands (δ_R and α_R), the optimal lag-length (κ^*), and the percentage of times the series spends in each of the intervention regimes.

Table 3. Duration and loss estimates

	$MaxD_L$	$MaxD_H$	$AveD_L$	$AveD_H$	$CumL_L$	$CumL_H$	$AveL_L$	$AveL_H$
Advanced	4.23	4.08	1.59	1.62	0.10	0.05	0.02	0.01
G7	4.57	4.00	1.61	1.60	0.06	0.04	0.01	0.02
Other	3.83	4.17	1.57	1.64	0.13	0.06	0.02	0.01
Developing	4.15	4.46	1.53	1.72	0.32	0.15	0.04	0.02
Asia	4.60	3.60	1.65	1.51	0.26	0.09	0.03	0.02
WH	3.88	5.00	1.46	1.86	0.35	0.18	0.04	0.03
Overall	4.19	4.27	1.56	1.67	0.21	0.10	0.03	0.02

Note: Let subscript R depict the alternative regimes, with L corresponding to over-depreciation and H to over-appreciation. $MaxD_R$ shows average maximum duration of excess deviations on each side of the band (number of periods); $AveD_R$ is the average duration per spell of excess deviation, across countries for each regime; $CumL_R$ is the cumulative excess deviation (area between the tolerance margin and the observed realizations when the band is crossed); and $AveL_R$ is the average excess deviation, across countries for each regime.

probably closely watched bands. The differences are more marked in subregions. For over-depreciations, the other (non-G7) advanced countries have the fastest response (0.62), Asia the slowest (0.45); for over-appreciations, the countries of WH have the slowest response (0.38), Asia the fastest (0.63). If this design of the thresholds reflects a measure of relative tolerance for these exchange rate variations, then the results suggest that G7 and Asian countries exercise greater caution against over-appreciations.

Asymmetry of response: On average, both advanced and developing countries display asymmetrical response to changes in the real exchange rates, with G7 (0.55 vs. 0.51) and Asia (0.63 vs. 0.45) placing greater weight on recent developments relating to appreciations while predicting the tolerance margin. The opposite is true for the other advanced (0.51 vs. 0.62) and WH (0.38 vs. 0.53) countries, which react more strongly to developments relating to over-depreciations.

Maximum durations of spells: These are somewhat longer for over-appreciations in WH and other advanced countries but longer for over-depreciations for G7 and Asian countries. As in the other statistics, the subgroups reveal differences. The maximum duration for the G7 occurs in the lower regime (4.6 months), but in the upper regime for the other advanced countries (4.2 months). Similarly, the maximum duration for Asia is in the lower regime (4.6 months), but in the upper regime for the WH countries (5 months).

Average duration of spells: In general, the average duration of periods between threshold crossings is somewhat higher for appreciations than for depreciations. The G7 countries have equal durations for both types of deviations, while Asian countries having higher durations for over-depreciations. The WH countries have the largest difference in average duration. Given the difference in response towards depreciation and appreciation deviations of the subgroups, the evidence of duration is probably informative about the speed or effectiveness of the policy measures used to reverse deviations from forecasts.

Asymmetry in duration of deviations: Average durations in the lower regime exceed that in the upper regime in 38% of both other advanced countries and developing countries, but these percentages mask inter-regional differences. Specifically, average duration in the lower regime is greater than the average duration in the upper regime in 57% of G7 and 80% of Asian countries, compared to 17% of other advanced countries and 13% for WH countries.

Frequency of thresholds being crossed: For developing countries, there is a tendency for more observations to lie in the upper regime (30% vs. 26%), more so for WH countries; however, with longer average durations for over-appreciations, the lower regime is characterized with a higher frequency of threshold crossings. The advanced economies experience similar frequency and duration of deviations on both sides of the bands, though slightly less pronounced. The observation that the developing countries sampled seem to watch their depreciation thresholds more closely is consistent with their recording more deviations in the upper regime and having a higher frequency of crossings in the lower regime.

Cumulative excess deviation per spell: If we define the cumulated difference between the actual exchange rate change and the expected change for the duration of a crossing as an excess deviation measure, we find that, for all groups, the excess deviation for a depreciation spell (crossing beyond the lower threshold) is twice as large as that for an appreciation spell (crossing beyond the upper threshold). The overall average is 0.21 in the lower regime and 0.10 in the upper regime. For the Asian countries, the excess deviation per depreciation spell is three times higher than that per appreciation spell; in contrast, the factor is 1.5 for G7 countries. Further, the excess deviation per depreciation and appreciation spells is about three times higher for developing countries relative to advanced countries.

Average excess deviation per spell: We calculate the average excess deviation per spell and find that, for both the advanced and developing countries, average excess deviation for depreciations are about twice that for appreciations. Also, average excess deviations per spells of depreciation and appreciation for developing countries is about twice that for advanced countries. But there are differences among subgroupings. For the advanced countries, the average excess deviation per appreciation spell is twice that of a depreciation spell in the G7; in contrast, the average excess deviation per depreciation spell is twice that of an appreciation spell in the other (non-G7) advanced countries. The average excess deviation for depreciations in the developing countries is four times that of the G7 countries; on the other hand, the average excess deviation for appreciations in the developing countries is the same as that for the G7 countries. We compare average excess deviation per spell in the upper and lower regimes and find that the average excess deviation per spell in the lower regime is greater than the average excess deviation per spell in the upper regime in all of the developing countries, compared to 57% of G7 and 83% of other advanced countries.

Parameter estimates. Tables 4–6 summarize the TAR estimates and the Wald tests. For the unrestricted TAR model, $\rho_L > \rho_H$ for developing countries and $\rho_H > \rho_L$ for advanced countries, consistent with faster reversion in developing countries for over-depreciations and faster reversion in advanced countries for over-appreciations. For G7 and Asian countries, only $\rho_H < 0$; on the other hand, $\rho_L > 0$ and larger than ρ_H for WH countries. In the corridor regime, all reversion coefficients are negative. For the TARurCor model, $|\rho_L| > |\rho_H|$ for WH countries, with approximate equality for G7 and Asian countries. Except for the other advanced countries, for which only ρ_H is negative, the reversion coefficients are negative and larger for depreciations relative to appreciations and for developing countries relative to advanced countries. As suggested by Caner and Hansen (2001), our tests are likely to be more powerful for WH, given the size of the threshold effects.

On the basis of the estimated TAR models, we calculate Wald statistics to test for threshold effects and/or unit roots. The tests measure whether the DGP under the null produces a residual variance that is significantly larger than the residual variance obtained from the fit of the alternative hypothesis, in our case the unrestricted TAR specification. Table 6 shows the percentage of countries for which the various null hypotheses are plausible (see Table 7 for details). These statistics are based on estimated unconstrained

Table 4. Summary reversion coefficients

	Linear	Unrestricted TAR			TARurCor	
	ρ_{LIN}	ρ_L	ρ_C	ρ_H	ρ_L	ρ_H
Advanced	−0.0213	0.0000	−0.0218	0.0030	−0.0076	−0.0140
G7	−0.017	0.0042	−0.0153	−0.006	−0.019	−0.010
Other	−0.0257	−0.0049	−0.0294	0.0140	0.0059	−0.0191
Developing	−0.0302	−0.0185	−0.0101	−0.0140	−0.0391	−0.0212
Asia	−0.0104	0.0080	−0.0097	−0.0075	−0.0167	−0.0119
WH	−0.0426	−0.0351	−0.0104	−0.0180	−0.0531	−0.0270
Overall	−0.0257	−0.0093	−0.0160	−0.0055	−0.0234	−0.0176

Note: Subscripts depict the alternative regimes, with L corresponding to over-depreciation, H to over-appreciation and C to the corridor. LIN refers to the linear model.

Table 5. TAR parameters of interest

	Linear		TAR						TARUR			TARURCOR				
	Conditional mean	Reversion	Conditional mean			Reversion			Conditional mean			Conditional mean			Reversion	
	μ	ρ	μ_L	μ_C	μ_U	ρ_L	ρ_C	ρ_U	μ_L	μ_C	μ_U	μ_L	μ_C	μ_U	ρ_L	ρ_U
Advanced *G7*																
Canada	−0.0008	−0.0049	−0.0039	0.0003	0.0005	−0.0462	0.0178	−0.0326	−0.0039	0.0003	0.0005	−0.0146	0.0043	0.0122	−0.0138	−0.0027
France	−0.0007	−0.0252	−0.0032	0.0003	0.0011	−0.0384	−0.0008	−0.0604	−0.0032	0.0003	0.0011	−0.0086	0.0024	0.0083	−0.0298	−0.0147
Germany	−0.0007	−0.0266	−0.0038	0.0007	0.0042	0.0161	−0.0426	0.0387	−0.0038	0.0007	0.0042	−0.0095	0.0034	0.0112	0.0004	−0.0064
Italy	0.0001	−0.0165	−0.0047	0.0017	0.0045	0.0150	−0.0248	0.0198	−0.0047	0.0017	0.0045	−0.0123	0.0043	0.0105	−0.0245	−0.0307
Japan	0.0014	−0.0144	−0.0062	0.0044	0.0125	0.0163	−0.0098	−0.0029	−0.0062	0.0044	0.0125	−0.0241	0.0120	0.0301	−0.0147	0.0078
United Kingdom	0.0006	−0.0249	−0.0062	0.0035	0.0075	0.0233	−0.0211	−0.0224	−0.0062	0.0035	0.0075	−0.0174	0.0085	0.0185	−0.0202	−0.0206
United States	0.0010	−0.0097	−0.0052	0.0036	0.0098	0.0430	−0.0260	0.0156	−0.0052	0.0036	0.0098	−0.0165	0.0084	0.0176	−0.0315	0.0007
Other																
Australia	−0.0012	−0.0130	−0.0096	0.0016	0.0042	0.0169	−0.0401	0.0494	−0.0096	0.0016	0.0042	−0.0292	0.0083	0.0196	0.0063	−0.0266
Belgium	−0.0007	−0.0250	−0.0039	0.0010	0.0027	−0.0187	−0.0219	0.0193	−0.0039	0.0010	0.0027	−0.0071	0.0028	0.0079	−0.0073	0.0035
Israel	0.0009	−0.0410	−0.0005	0.0013	0.0059	−0.1095	−0.0227	0.0162	−0.0005	0.0013	0.0059	−0.0240	0.0076	0.0179	−0.0250	−0.0144
Korea	−0.0007	−0.0298	−0.0150	0.0042	0.0086	0.0380	−0.0313	−0.0047	−0.0150	0.0042	0.0086	−0.0233	0.0070	0.0186	0.0760	−0.0644
New Zealand	−0.0005	−0.0296	−0.0064	0.0010	0.0041	0.0433	−0.0483	0.0158	−0.0064	0.0010	0.0041	−0.0252	0.0059	0.0173	0.0191	−0.0138
Spain	−0.0004	−0.0157	−0.0053	0.0013	0.0050	0.0007	−0.0122	−0.0121	−0.0053	0.0013	0.0050	−0.0117	0.0034	0.0105	−0.0335	0.0011
Developing *Asia*																
India	−0.0030	−0.0031	−0.0093	−0.0006	0.0036	0.0028	−0.0018	−0.0034	−0.0093	−0.0006	0.0036	−0.0225	0.0044	0.0173	0.0112	−0.0074
Indonesia	−0.0043	−0.0106	−0.0182	0.0020	0.0069	0.0340	0.0010	−0.0502	−0.0182	0.0020	0.0069	−0.0454	0.0147	0.0389	−0.0254	−0.0357
Malaysia	−0.0014	−0.0072	−0.0058	0.0002	0.0047	−0.0182	−0.0040	−0.0069	−0.0058	0.0002	0.0047	−0.0182	0.0054	0.0138	0.0035	−0.0070
Philippines	−0.0012	−0.0201	−0.0152	0.0025	0.0087	−0.0152	−0.0282	0.0300	−0.0152	0.0025	0.0087	−0.0350	0.0079	0.0220	−0.0251	−0.0064
Thailand	−0.0011	−0.0111	−0.0042	0.0003	0.0110	0.0364	−0.0155	−0.0066	−0.0042	0.0003	0.0110	−0.0218	0.0080	0.0191	−0.0479	−0.0030
WH																
Argentina	−0.0003	−0.0213	−0.0063	0.0018	0.0073	0.1009	−0.0567	0.0833	−0.0063	0.0018	0.0073	−0.0614	0.0225	0.0356	0.0519	−0.0692
Brazil	−0.0012	−0.0410	−0.0132	0.0027	0.0102	−0.0765	−0.0145	−0.0218	−0.0132	0.0027	0.0102	−0.0472	0.0136	0.0325	−0.0081	−0.0517
Chile	−0.0024	−0.0146	−0.0113	0.0008	0.0078	−0.0643	0.0023	−0.0178	−0.0113	0.0008	0.0078	−0.0304	0.0073	0.0231	−0.0540	0.0013
Colombia	−0.0014	−0.0060	−0.0084	0.0013	0.0072	−0.0010	−0.0066	0.0012	−0.0084	0.0013	0.0072	−0.0236	0.0069	0.0190	−0.0010	−0.0117
Costa Rica	0.0003	−0.1757	−0.0102	0.0039	0.0075	−0.0732	0.0322	−0.1213	−0.0102	0.0039	0.0075	−0.0249	0.0089	0.0137	−0.1021	−0.0472
Mexico	0.0005	−0.0439	−0.0146	0.0032	0.0118	−0.0697	−0.0377	0.0022	−0.0146	0.0032	0.0118	−0.0656	0.0124	0.0233	−0.2031	−0.0191
Paraguay	−0.0027	−0.0195	0.0002	−0.0037	0.0021	−0.0252	−0.0115	−0.0265	0.0002	−0.0037	0.0021	−0.0455	0.0117	0.0291	−0.0632	0.0168
Uruguay	0.0002	−0.0184	−0.0120	0.0039	0.0111	−0.0715	0.0092	−0.0438	−0.0120	0.0039	0.0111	−0.0334	0.0106	0.0259	−0.0454	−0.0350

Note: Subscripts depict the alternative regimes, with L corresponding to over-depreciation, II to over-appreciation and C to the corridor. *LIN* refers to the linear.

Table 6. Summary of Wald tests

	Lin vs. TAR	LinUR vs. TAR	TARur vs. TAR	TARurCor vs. TAR
Advanced	0.00	0.00	0.08	0.23
G7	0.00	0.00	0.14	0.29
Other	0.00	0.00	0.00	0.17
Developing	0.00	0.00	0.15	0.69
Asia	0.00	0.00	0.20	0.60
WH	0.00	0.00	0.13	0.75
Overall	0.00	0.00	0.12	0.46

Note: Numbers are percentages of do not reject, based on Wald tests.

Table 7. Wald tests

	LIN vs. TAR		LINur vs. TAR		TARur vs. TAR		TARurCor vs. TAR	
	W1	UnC	W2	UnC	W3	UnC	W4	UnC
Advanced *G7*								
Canada	21.70	0.00	21.19	0.00	6.89	0.00	2.43	0.00
France	4.41	0.00	7.81	0.00	6.79	0.00	1.07	1.00
Germany	3.33	0.00	8.91	0.00	6.29	0.00	6.00	0.00
Italy	41.64	0.00	44.08	0.00	1.14	0.00	2.91	0.00
Japan	40.54	0.00	43.61	0.00	1.32	1.00	0.27	1.00
United Kingdom	30.49	0.00	34.05	0.00	1.99	0.00	0.44	0.00
United States	16.26	0.00	16.82	0.00	3.59	0.00	3.76	0.00
Other								
Australia	25.57	0.00	26.16	0.00	5.28	0.00	5.94	0.00
Belgium	16.00	0.00	24.26	0.00	10.07	0.00	1.90	0.00
Israel	25.20	0.00	29.86	0.00	12.38	0.00	0.25	1.00
Korea	11.66	0.00	16.02	0.00	1.10	0.00	1.81	0.00
New Zealand	24.83	0.00	28.56	0.00	4.54	0.00	4.59	0.00
Spain	20.94	0.00	24.28	0.00	1.56	0.00	0.39	0.00
Developing *Asia*								
India	4.29	0.00	4.13	0.00	2.34	1.00	0.88	1.00
Indonesia	219.80	0.00	220.51	0.00	11.28	0.00	1.14	1.00
Malaysia	102.20	0.00	102.66	0.00	2.90	0.00	0.90	1.00
Philippines	47.66	0.00	50.17	0.00	4.43	0.00	2.91	0.00
Thailand	115.48	0.00	115.71	0.00	1.14	0.00	0.63	0.00
WH								
Argentina	157.30	0.00	159.97	0.00	13.33	0.00	10.26	0.00
Brazil	61.80	0.00	67.33	0.00	6.55	0.00	0.81	1.00
Chile	49.92	0.00	53.69	0.00	17.24	0.00	1.09	1.00
Colombia	5.76	0.00	6.45	0.00	0.98	1.00	0.06	1.00
Costa Rica	750.42	0.00	889.37	0.00	12.08	0.00	0.07	1.00
Mexico	95.74	0.00	104.03	0.00	9.72	0.00	1.93	0.00
Paraguay	18.74	0.00	20.99	0.00	4.17	0.00	0.53	1.00
Uruguay	9.74	0.00	10.96	0.00	5.93	0.00	0.77	1.00

Note: W1 is a test for the existence of a threshold; W2 tests for a unit root when there is no threshold effect; W3 tests for a unit root in the presence of threshold effects; and W4 tests for a (partial) unit root only in the corridor regime. UNC indicates rejection (0) of null based on the unconstrained bootstrap critical values. We report absolute values but, in a few cases, we obtained small and negative statistics for W3 and W4, arising from the small sample adjustment to the variance under the null and the alternative.

bootstrap p-values, representing the percentage of Wald statistics calculated from the simulated data that exceed the Wald statistics calculated from the observed sample.

The results indicate an overwhelming rejection of the first three null hypotheses. The unrestricted TAR specification outperforms the benchmark stationary ergodic linear process. It is also preferred over both the linear nonstationary $I(1)$ specification, the p-values for which are obtained by constructing a bootstrap distribution that imposes an unidentified threshold effect, and the unit root TAR process.[22] Because the unidentified threshold model was less sensitive to nuisance parameters, Caner and Hansen (2001) recommend calculating p-values using the unidentified threshold bootstrap. The intermediate case, which we label as an identified threshold partial unit root process ($I(1)$ in corridor regime combined with an otherwise stationary TAR), yields different outcomes for advanced and developing countries. While the null is still rejected against the stationary ergodic TAR for most advanced countries, the developing countries do not reject the partial unit root TAR as their preferred specification. Thus, the partial unit root model could characterize the data dynamics for these countries.

3.2. STR estimates

Testing for linearity. The first step in estimating an STR model is to test for linearity against STR-type nonlinearity, which implies testing the null hypothesis $H_0 : \theta_2' = 0$ in equation (2). Under the null hypothesis, the parameters γ and c are not identified. The solution advocated by Luukkonen *et al.* (1988) and adopted by Teräsvirta (1994) is to replace the transition function by a suitable Taylor series approximation. We propose considering a third-order Taylor expansion of the transition function for the BSTR model.[23] Substituting

$$T_3 = F_t^*(z_t^d) + \sum_i \gamma_i \frac{\partial F_t^*(z_t^d)}{\partial \gamma_i} + \frac{1}{2!} \sum_i \sum_j \gamma_i \gamma_j \frac{\partial^2 F_t^*(z_t^d)}{\partial \gamma_i \partial \gamma_j} + \frac{1}{3!} \sum_i \sum_j \sum_k \gamma_i \gamma_j \gamma_k \frac{\partial^3 F_t^*(z_t^d)}{\partial \gamma_i \partial \gamma_j \partial \gamma_k} + R_3 \tag{6}$$

for the transition function in equation (2), with all terms evaluated at $\gamma_1 = \gamma_2 = 0$, yields an auxiliary regression:

$$\Delta y_t = \beta_0 x_{t-1} + \beta_1 x_{t-1} z_t^d + \beta_2 x_{t-1} (z_t^d)^2 + \beta_3 x_{t-1} (z_t^d)^3 + \eta_t \tag{7}$$

where

$$\eta_t = \theta' x_t R_3 + \varepsilon_t$$

and

$$\begin{aligned}
\beta_0 &= \underbrace{\phi' + \tfrac{1}{9}\theta'(c_1\gamma_1 - c_2\gamma_2)}_{I} + \underbrace{\theta'(\tfrac{1}{54} c_1^2\gamma_1^2 + \tfrac{2}{27} c_1 c_2 \gamma_1 \gamma_2) + \tfrac{1}{54} c_2^2 \gamma_2^2)}_{II} + \underbrace{\tfrac{1}{162}\theta'(c_2^3\gamma_2^3 - c_1^3\gamma_1^3)}_{III} \\
\beta_1 &= \tfrac{1}{9}\theta' \underbrace{(-\gamma_1 + \gamma_2)}_{I} + \underbrace{\tfrac{1}{27}\theta'(-c_1\gamma_1^2 - 2c_1\gamma_1\gamma_2 - c_2\gamma_1\gamma_2 - c_2\gamma_2^2)}_{II} + \underbrace{\tfrac{1}{54}\theta'(c_1^2\gamma_1^3 - c_2^2\gamma_2^3)}_{III} \\
\beta_2 &= \underbrace{\tfrac{1}{54}\theta'\gamma_1^2 + \theta'(\tfrac{2}{27}\gamma_1\gamma_2 + \tfrac{1}{54}\gamma_2^2)}_{II} + \underbrace{\tfrac{1}{54}\theta'(c_2\gamma_2^3 - c_1\gamma_1^3)}_{III} \\
\beta_3 &= \underbrace{\tfrac{1}{162}\theta'(\gamma_1^3 - \gamma_2^3)}_{III}
\end{aligned} \tag{8}$$

Since β_3 is not dependent on c_1 or c_2, and all $\beta_j = 0, j = 1, \ldots, 3$, for $\gamma_1 = \gamma_2 = 0$, it follows that, conditional on rejecting linearity ($\beta_j \neq 0, j = 1, \ldots, 3$), a do not reject of the hypothesis $\beta_3 = 0$ indicates $\gamma_1 = \gamma_2$ and suggests a symmetric three-regime STR model. If the hypothesis of symmetry is not rejected, tests exist for choosing among logistic and exponential STAR models (see Teräsvirta, 1998; Escribano and Jorda, 1999).

Parameter estimates. Table 10 includes results of a linearity test against smooth transition alternatives. In executing the linearity tests, the lag length p was chosen based on the Akaike information criterion applied to a linear AR for Δy_t. The first and second blocks report p-values of F-tests for the auxiliary regression (7) with Δy_{t-1} and *time* as the transition variables, respectively.[24]

With the exception of the United States and India, the linearity test results provide uniformly strong evidence against linearity in favour of STR-form nonlinearity for a number of transition variables considered. The tests show stronger rejection of linearity (across potential transition variables) for developing countries relative to advanced countries; linearity is rejected against smooth transition time variation only in developing countries. For the BSTR alternative, the hypothesis of symmetry in regime transition, $(F_3): \beta_3 = 0$ in equation (7), is rejected for almost half of the countries, less so for the G7 countries. Using the lag length chosen by AIC for the corresponding linear AR specifications, and with the choice of $\Delta \ln y_{t-1}$ as transition variable (linearity test result), the appropriate BSTAR models are estimated and their results reported in Tables 8 and 9.[25] Following Teräsvirta (1998), the transition parameter was standardized through division by its sample variance and the initial value of γ, the adjustment speed parameter, was fixed at 1 for the estimation algorithm.

Tables 8 and 9 show that the threshold range is wider in developing countries and the speed of adjustment is greater at the lower threshold (γ_L); in fact, $\gamma_L > \gamma_H$ in 62% of countries. Comparing conditional and unconditional means, we find that in 96% of cases the addition of the nonlinear component to the model indicates reversion to the mean. The duration estimates indicate a higher probability of being in the upper regime; exceptions among advanced countries are Australia, Canada, Germany, and Japan. As an interpretational example, we reproduce below (equation (9)) the BSTAR result for Canada. The lower and upper thresholds are at -0.6% and 1%, respectively, indicating a higher threshold tolerance for appreciations. The reversion coefficient, which is significantly different from zero, interacts with the transition function, indicating different reversion speeds, depending on the value of the transition function. The speeds of adjustment are 0.33 from the lower to the middle regime and 2.47 between the middle and upper regimes, indicating a quicker move between the corridor and appreciation regimes than between the depreciation and corridor regimes.

Estimated parsimonious BSTAR model for Canada ($p = 2$):

$$\Delta y_t = \underset{(21.3)}{0.10} - \underset{(2.77)}{0.80}\,\Delta y_{t-2} + \left\{ \underset{(4.04)}{0.20} - \underset{(3.73)}{0.04}\,y_{t-1} + \underset{(2.98)}{0.32}\,\Delta y_{t-1} - \underset{(3.06)}{1.46}\,\Delta y_{t-2} \right\} F(\Delta y_{t-1})$$

$$F(\Delta y_{t-1}) = \frac{\exp\,[-(0.33/\sigma_{\Delta y})(\Delta y_{t-1} - (-0.006))] + \exp\,[(2.47/\sigma_{\Delta y})(\Delta y_{t-1} - 0.01)]}{1 + \exp\,[-(0.33/\sigma_{\Delta y})(\Delta y_{t-1} - (-0.006))] + \exp\,[(2.47/\sigma_{\Delta y})(\Delta y_{t-1} - 0.01)]} \tag{9}$$

Table 10 also presents results on symmetry and encompassing. The third block reports tests for the feasibility of regime reduction (from three to two regimes), that is $c_L = c_H$, and asymmetry ($\gamma_L = \gamma_H$).[26] The fourth block reports encompassing tests of the linear model relative to the nonlinear model. The final column reports the ratio of the variance of the STR residuals to variance of the linear residuals. We find ample evidence consistent with three-regime switching regressions and asymmetric adjustment speeds between regimes: $c_L \neq c_H$ in 81% of cases, and $\gamma_L \neq \gamma_H$ in 58% of countries. Among G7 countries, we cannot reject symmetry for the two major currency countries, Germany and Japan. Further, for these two countries durations in each regime are approximately equal, probably reflecting the market microstructure of these advanced economies. The results show that $c_L = c_H$ in France (among advanced countries) and in

Table 8. BSTAR summary coefficients

	c_L	c_H	γ_L	γ_H	M_{Lin}	M_{NonLin}	$M_{\Delta y}$	Reversion
Advanced	−0.012	0.009	0.789	0.820	0.022	−0.002	0.000	0.917
G7	−0.010	0.007	0.707	1.064	0.007	0.000	0.000	1.000
Other	−0.013	0.010	0.872	0.577	0.037	−0.005	0.000	0.833
Developing	0.006	0.067	0.960	0.696	0.016	−0.002	−0.001	1.000
Asia	0.037	0.162	0.763	0.502	0.002	−0.002	−0.002	1.000
WH	−0.012	0.007	1.083	0.818	0.025	−0.002	−0.001	1.000
Overall	−0.002	0.039	0.878	0.756	0.019	−0.002	−0.001	0.960

Note: c_L, c_H are threshold values, γ_L, γ_H are speeds of regime transition, and reversion shows the percentage of times the difference between the conditional mean from the nonlinear model (M_{NonLin}) and the unconditional mean ($M_{\Delta y}$) is less than the corresponding difference for the linear model (M_{Lin}).

Table 9. BSTR parameters of interest

	Parameters				Conditional mean		Mean	Duration		
	c_L	c_H	γ_L	γ_H	M_{Lin}	M_{NonLin}	$M_{\Delta y}$	d_L	d_C	d_H
Advanced *G7*										
Canada	−0.006	0.010	0.333	2.468	−0.0121	−0.0007	−0.0006	1.58	1.96	1.49
France	−0.010	0.006	0.160	1.298	0.0029	−0.0007	−0.0007	1.28	3.84	1.34
Germany	−0.010	0.004	1.053	0.682	−0.0012	−0.0007	−0.0007	1.76	1.88	1.78
Italy	−0.011	0.001	0.847	0.369	0.0401	0.0002	0.0000	1.47	2.35	2.66
Japan	−0.010	0.019	1.123	1.148	0.0072	0.0016	0.0017	1.63	2.00	1.50
United Kingdom	−0.013	0.003	0.726	0.417	0.0056	0.0007	0.0010	1.57	2.27	1.73
United States										
Other										
Australia	−0.017	0.013	1.131	0.425	0.0096	−0.0012	−0.001	1.75	2.24	1.50
Belgium	−0.013	0.002	0.536	0.744	0.0098	−0.0008	−0.0008	1.57	3.85	1.77
Israel	−0.003	0.028	1.275	0.222	−0.0001	−0.0234	0.0009	2.11	2.80	1.00
Korea	−0.017	0.011	0.775	0.702	0.2011	−0.0014	−0.0008	1.42	2.72	1.77
New Zealand	−0.018	0.005	0.689	0.788	−0.0007	−0.0002	−0.0004	1.45	2.23	1.96
Spain	−0.012	0.001	0.824	0.578	0.0036	−0.0004	0.0000	1.65	2.42	2.00
Developing *Asia*										
India	0.271	0.830	1.179	0.918	0.0125	−0.0034	−0.0029	1.69	2.67	1.17
Indonesia	−0.018	−0.011	0.685	0.146	−0.0126	−0.0040	−0.0035	1.44	1.13	3.92
Malaysia	−0.010	0.008	0.725	0.418	0.0419	−0.0020	−0.0014	1.63	2.65	1.56
Philippines	−0.017	−0.001	0.738	0.497	−0.0194	−0.0015	−0.0010	1.61	1.28	3.08
Thailand	−0.042	−0.015	0.488	0.531	−0.0143	−0.0006	−0.0009	1.29	1.29	9.88
WH										
Argentina	0.008	−0.023	1.778	0.323	−0.0053	−0.0001	0.0003	3.16	1.00	8.50
Brazil	−0.022	0.016	0.608	0.825	−0.0473	−0.0011	−0.0006	1.57	2.57	1.69
Chile	−0.011	0.017	1.107	0.529	0.0361	−0.0026	−0.0018	1.69	2.38	1.55
Colombia	−0.024	0.006	0.624	1.437	−0.0062	−0.0014	−0.0014	1.42	2.37	2.27
Costa Rica	−0.010	0.009	0.907	1.016	−0.1565	−0.0009	−0.0012	1.59	2.67	2.03
Mexico	−0.009	0.012	1.162	0.836	0.1669	−0.0020	0.0008	1.75	2.13	2.02
Paraguay	−0.018	0.011	1.018	0.923	−0.0872	−0.0027	−0.0028	1.53	2.56	1.86
Uruguay	−0.013	0.009	1.463	0.654	0.2983	−0.0032	0.0004	1.30	2.00	1.88

Note: c_L, c_H are threshold values, γ_L, γ_H, are speeds of regime transition, M_{Lin} and M_{NonLin} are conditional means from the linear and nonlinear models, $M_{\Delta y}$ is the unconditional mean. The parameters determine the transition function of the BSTR model $\Delta y_t = \theta_1' x_{t-1} + \theta_2' x_{t-1} F(z_t^d; \gamma, c) + \mu_t$, where $F_t(\gamma_L, c_L, \gamma_H, c_{H,}; z_t^d) = \dfrac{\exp[-\gamma_L(z_t^d - c_L)] + \exp[\gamma_H(z_t^d - c_H)]}{1 + \exp[-\gamma_L(z_t^d - c_L)] + \exp[\gamma_H(z_t^d - c_H)]}$, $\gamma_L, \gamma_H > 0$; $c_L < c_H$.

Indonesia, Philippines, Thailand and Argentina (among developing countries). Only Thailand does not reject both symmetry and adequacy of two regimes.

The results of the encompassing tests, based on the minimal nesting model (MNM) framework, indicate that the linear model does not encompass the nonlinear alternative while the nonlinear BSTR models encompass the corresponding linear models for all countries. Although the rich parameterization in an MNM framework is believed to endanger the convergence properties in tests of parsimonious encompassing (the BEGS algorithm may either not converge or converge to a local minimum), we did not encounter any convergence problems; in fact, the smooth convergence found suggests that the parameter estimates are very close to their optimal values. In terms of variance reduction, the largest improvements occur for the developing countries.

3.3. *MSM estimates*

Table 11 shows that an initial test of linearity versus nonlinearity of a Markov-switching form rejects the linear specification. The results for a Markov-switching intercept and autoregressive (MSIA) specification are reported in Tables 12 and 13.

Table 10. Linearity, symmetry, and encompassing tests

	Linearity				Symmetry		Encompassing		
	Δy_{t-1}		Time						
	F_{Lin}	F_3	F_{Lin}	F_3	$c_L = c_H$	$\gamma_L = \gamma_H$	*M_Lin*	*M_NL*	*V_NL/V_Lin*
Advanced *G7*									
Canada	0.018	0.161	0.102	0.419	0.017	0.000	0.000	0.818	0.940
France	0.035	0.096	0.251	0.105	0.165	0.000	0.000	1.000	0.956
Germany	0.082	0.575	0.420	0.228	0.000	0.349	0.000	1.000	0.964
Italy	0.000	0.713	0.431	0.247	0.000	0.001	0.000	0.956	0.842
Japan	0.000	0.014	0.185	0.955	0.000	0.909	0.000	0.136	0.858
United Kingdom	0.003	0.105	0.222	0.426	0.000	0.011	0.000	0.713	0.862
United States	0.685	0.257	0.111	0.439					
Other									
Australia	0.099	0.501	0.078	0.563	0.000	0.064	0.026	1.000	0.962
Belgium	0.000	0.000	0.666	0.677	0.001	0.221	0.000	0.777	0.918
Israel	0.003	0.077	0.461	0.681	0.000	0.008	0.000	0.637	0.937
Korea	0.000	0.000	0.419	0.909	0.000	0.709	0.038	1.000	0.856
New Zealand	0.001	0.051	0.521	0.379	0.016	0.695	0.000	1.000	0.952
Spain	0.000	0.000	0.440	0.689	0.000	0.001	0.000	1.000	0.895
Developing *Asia*									
India	0.779	0.695	0.068	0.410	0.000	0.291	0.000	0.419	0.957
Indonesia	0.000	0.000	0.077	0.436	0.677	0.000	0.000	0.153	0.659
Malaysia	0.000	0.000	0.138	0.504	0.000	0.008	0.000	1.000	0.691
Philippines	0.000	0.013	0.015	0.032	0.169	0.091	0.000	1.000	0.894
Thailand	0.000	0.000	0.050	0.521	0.181	0.656	0.000	1.000	0.850
WH									
Argentina	0.000	0.000	0.009	0.011	0.149	0.038	0.000	0.991	0.789
Brazil	0.000	0.006	0.002	0.087	0.001	0.429	0.019	0.370	0.887
Chile	0.000	0.021	0.000	0.000	0.000	0.005	0.000	1.000	0.794
Colombia	0.055	0.698	0.056	0.086	0.005	0.001	0.001	1.000	0.951
Costa Rica	0.000	0.000	0.000	0.000	0.016	0.544	0.000	0.883	0.706
Mexico	0.000	0.000	0.000	0.034	0.001	0.206	0.000	0.834	0.734
Paraguay	0.038	0.213	0.055	0.177	0.023	0.771	0.000	0.711	0.951
Uruguay	0.000	0.000	0.029	0.002	0.000	0.047	0.009	0.719	0.755

Note: For the linearity test (Δy_{t-1} and time as transition variables) we report: (F_{Lin}) $H_0^{LIN}: \beta_1 = \beta_2 = \beta_3 = 0$ and (F_3)$H_0: \beta_3 = 0$. The encompassing tests are calculated by estimating by NLLS an MNM form equation containing all of the explanatory variables for both models under consideration and then testing the restrictions necessary to obtain each model through F-tests. Subscripts Lin and NL refer to linear and nonlinear, respectively. V is variance. Numbers in symmetry and encompassing columns are p-values.

We find qualitatively similar results to the TAR process. The G7 and Asian countries have higher reversion coefficients in the upper regime, while other advanced and WH countries have larger reversion coefficients in the lower regime. Differences in the reversion coefficients tell only part of the story, as the sum of the coefficients on the lagged dependent variables also varies significantly across regimes and countries, indicating large differences in serial correlation properties of the series. The reversion coefficients in the upper and lower regimes are also unequal, suggesting asymmetrical adjustment. We report conditional means for the three regimes, but note that they are not strictly comparable across the three classes of models estimated. This is because the predicted value from the MSIA specification is a weighted average of all three regimes, in contrast to the regime-specific conditional means obtained in the TAR specifications, and the specific weights depend on the probability of being in each of the regimes at that time period.

We examine the transition probabilities p_{jj}, $j = 1, 2, 3$, for evidence of persistence (Table 13). The probability of remaining in regime 2 at time t, given that the process was in regime 2 at time $t-1$ is

Table 11. Linearity vs. MSM nonlinearity

G7	Canada	France	Germany	Italy	Japan	UKD	USA	
F-test	33.58	62.14	22.82	189.4	73.19	71.74	37.61	
p-value	0.002	0.000	0.004	0.000	0.000	0.000	0.000	
Other	Australia	Belgium	Israel	Korea	New Zealand	Spain		
F-test	86.13	117.4	78.6	259.7	74.42	124.1		
p-value	0.000	0.000	0.000	0.000	0.000	0.000		
Developing Asia	India	Indonesia	Malaysia	Philippines	Thailand			
F-test	121.7	506.6	246.5	185.7	331.4			
p-value	0.000	0.000	0.000	0.000	0.000			
WH	Argentina	Brazil	Chile	Colombia	Costa Rica	Mexico	Paraguay	Uruguay
F-test	347.5	259.3	219.7	50.16	514.0	437.8	147.2	364.3
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: UKD is United Kingdom and USA is United States.

Table 12. MSIA summary coefficients

	ρ_1	ρ_2	ρ_3	R_1	R_2	R_3
Advanced	−0.115	−0.004	−0.099	−0.020	−0.003	0.009
G7	−0.005	0.001	−0.064	−0.012	−0.000	0.013
Other	−0.028	−0.034	0.009	−0.028	−0.005	0.003
Developing	−0.037	−0.010	−0.037	−0.080	−0.007	0.008
Asia	−0.090	−0.004	−0.131	−0.053	−0.016	−0.007
WH	−0.084	−0.009	−0.063	−0.099	−0.001	0.020
Overall	−0.049	−0.011	−0.059	−0.048	−0.005	0.009

Note: ρ_1 are coefficients of y_{t-1} in $\Delta y_t = \sum_{R=1}^{m}\left(\alpha_R + \rho_R y_{t-1} + \sum_{i=1}^{k}\beta_{iR}\Delta y_{t-1}\right)\Pr(S_t = R \mid \Delta\tilde{y}_{t-1}) + \sigma v_t$ (equation (8)); R_i are conditional means.

uniformly higher across all groups, with developing countries having higher probabilities than advanced countries. It is also clear that the probability of remaining in the lower regime is uniformly less than that of remaining in the upper regime, although country-specific differences exist. For almost all countries, the probability of remaining in the upper regime is greater than that of remaining in the lower regime, suggesting an aversion to depreciations. Exceptions are Germany and Spain among advanced countries and Indonesia among developing countries; for the United States and Brazil, the probabilities of remaining in the upper and lower regimes are approximately equal. For developing countries, the probability of being in the lower regime is dominated by that of being in the upper regime; in contrast, two of the three major currency countries (Germany and the United States) have higher probabilities of being in the lower than the upper regime. Relative to the probabilities implied from the TAR framework (observable switching variable), the MSIA specification has higher and more variable probabilities associated with the middle regime.

3.4. Tests of model evaluation

We evaluate the performance of the models, using several measures. We consider a test of remaining nonlinearity based on Hamilton's (2001) general linearity test, Li's (1995) density-based nonparametric test,

Table 13. MSIA probabilities and duration estimates

	Transition probabilities			Unconditional probabilities			Average duration		
	p_{11}	p_{22}	p_{33}	P_{r1}	P_{r2}	P_{r3}	d_1	d_2	d_3
Advanced	0.36	0.73	0.63	0.15	0.54	0.31	1.99	10.10	5.47
G7	0.44	0.71	0.58	0.21	0.60	0.18	2.43	9.46	3.44
Other	0.26	0.76	0.68	0.07	0.47	0.46	1.47	10.84	7.84
Developing	0.29	0.85	0.61	0.07	0.71	0.23	1.52	13.50	3.43
Asia	0.27	0.92	0.54	0.05	0.81	0.14	1.41	18.38	3.06
WH	0.31	0.80	0.66	0.07	0.64	0.29	1.60	10.01	3.69
Overall	0.33	0.79	0.62	0.11	0.62	0.27	1.76	11.73	4.49

Note: Let $j = 1,2,3$ denote the alternative regimes. Then, p_{jj} indicates the probability of being in regime j given that we were in regime j the previous period; Pr_j is the unconditional probability of being in regime j; d_j is the average duration in regime j.

and the generalized conservative test framework proposed by BNP (2002), which evaluates the properties of interest implied by the simulated models against the empirical properties of the data.[27] We do not control for alternative models' sensitivity to outliers or extreme observations.

Applying Hamilton's (2001) generalized test for nonlinearity to the residuals of our estimated models indicates that the TARur model accounts for the nonlinearity in the G7 countries (Table 14). The incidence of remaining nonlinearity is 17% for the other advanced countries and 39% for the developing countries. The TAR corridor model shows remaining nonlinearity in about half of the countries, suggesting that specification is less adequate as a characterization of the data dynamics. The BSTR specification has the lowest incidence of remaining nonlinearity, with zero incidence for the G7 countries and 12.5% for WH. In contrast, the MSIA performs poorly.

Li's test is based on matching the empirical density of the observed series (estimated through a nonparametric kernel estimator) to the density implied by the simulated model, based on the estimates of each specification considered above. The reported statistics are meant to be interpreted in relative terms across the models as representations of a measure of deviations between the two densities. In terms of relative performance, the BSTR outperforms the other nonlinear models in achieving the closest match between the two densities. In about two-thirds of cases, the nonlinear models (TAR and BSTAR) outperform the linear models (LIN and LINur), confirming that our nonlinear specifications are more adequate characterizations of the data generating process than the linear models.

For BNP (2002), we consider tests for the first two moments (mean and variance), the interquartile range (the middle 50% of the observations), and measures of asymmetry and persistence. For asymmetry and persistence, we measure how well the data simulated under the estimated models replicates the features of EGARCH-asymmetry and GARCH-persistence in the conditional variance of the empirical sample. The tests are based on the comparison of the series' empirical density, estimated nonparametrically, and the density implied by each of the models, obtained from simulations using 1000 replications as the trimming margin. In calculating the Newey–West standard errors, nine lags were used to account for possible serial correlation. The estimated statistics show that, in terms of relative performance, the corridor unit root model (TARurCor) performs the least well in matching the two densities. The unrestricted (stationary) threshold model performs slightly better than the threshold model with a unit root in each regime (TARur). The performances of the linear and linear unit root models are similar, probably indicating a near unit root estimate. In contrast to the Wald tests, the BNP tests are less discriminating, because they are conservative and therefore under-reject. This suggests that when they do reject, there is an extremely strong case for rejection and any other less conservative test would also reject. More importantly, they provide critical information on the exact moment-based measure that is responsible for misspecification in the estimated model.

Although most models perform well in matching the mean and variance of the data, their ability to replicate the interquartile range and to a lesser extent asymmetry and persistence is less impressive, especially in developing countries. In about 50% of advanced countries, all the characteristics tested are replicated by at least one model; further, the linear models are also capable of replicating some characteristics of the data densities. A result that seems consistent across countries is that, among nonlinear models, the BSTAR model comes closest to the unconditional mean and the asymmetry-based measure; among the linear models, the nonstationary model outperforms its stationary counterpart.

4. CONCLUSIONS

This paper addresses some key questions in the PPP debate: (1) are deviations from PPP stationary; (2) are linear specifications appropriate; (3) is adjustment towards PPP symmetric from above and below; and (4) which nonlinear models more adequately characterize the process generating real exchange rates? Our results indicate that the notion of a unit root in real effective exchange rates is not robust to nonlinear specifications. Second, the adjustment dynamics of real exchange rates is not symmetric and that asymmetry differs across countries. Third, a three-regime smooth transition autoregressive model with asymmetric speeds of adjustment between regimes performs best, but not across all countries. While our Markov-switching model performed the least well among the models considered, we caution that the specification used can be extended in a number of directions, which could improve performance (see Hamilton and Raj, 2002).

The evidence in this paper includes a number of empirical characteristics that theory models should seek to explain. Of particular interest is the finding of asymmetrical adjustment, because different durations and frequencies of threshold crossings imply different degrees to which countries are prone to macroeconomic consequences of real exchange rate misalignments. The finding of asymmetry also suggests that transactions costs alone cannot explain the dynamics of real exchange rates;[28] in contrast, asymmetry is not inconsistent with an intervention interpretation of the dynamics of real exchange rates. Our results relate to those of Taylor (2004), who uses a Markov-switching model to show that the probability of switching between stable and unstable regimes depends nonlinearly on the amount of intervention, the degree of misalignment, and the duration of the regime. In particular, the probability of switching from the unstable to the stable regime increases as the real exchange rate deviates farther from its equilibrium level and the size of the misalignment grows, that is, real exchange rates are more likely to mean revert the farther they are from the equilibrium rate. Asymmetry also holds on a cross-section basis. Using the results from identical TAR models for an expanded set of 35 countries, for which both debt and openness data were available, Leon and Najarian (2003) found a positive correlation between average openness[29] and average duration for over-appreciations but no correlation between average openness and average duration for over-depreciations; similarly, they found a positive correlation between the average debt-to-GDP ratio and the average excess deviation (as defined in this paper) for over-depreciations but no correlation between the debt-to-GDP ratio and the excess deviation for over-appreciations. The implication that openness may be related to duration of over-appreciation misalignments but debt ratios are related to excess deviations of over-depreciations merits further research.[30] Second, the inability of the nonlinear models to explain all the

Table 14. Summary of Hamilton's nonlinearity test

	TAR	TARur	TARurCor	BSTR	MSIA
Advanced	0.154	0.077	0.462	0.167	0.615
G7	0.143	0.000	0.286	0.000	0.857
Other	0.167	0.167	0.667	0.333	0.333
Developing	0.385	0.385	0.462	0.231	0.750
Asia	0.200	0.400	0.400	0.400	1.000
WH	0.500	0.375	0.500	0.125	0.571
Overall	0.269	0.231	0.462	0.200	0.680

Note: Numbers are percentage of countries that reject the hypothesis of no remaining nonlinearity.

characteristics of the data examined indicates the limitations of current specifications and which issues/objectives they are capable of addressing; it also points to the need to develop specifications that account, at least, for higher moments of the data. A third implication of our work, meriting further research, relates to model selection and testing. Our research suggests that formal hypothesis testing would probably be more interpretable in the context of a set of models that are capable of replicating the same characteristics of the data.

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NOTES

1. Nonlinearities in exchange rates can also occur because of: (i) heterogeneity in agents' expectations, given different investment horizons, risk profiles and institutional constraints (Brock and Holmes, 1998; DeGrauwe and Grimaldi, 2002; Kilian and Taylor, 2002); and (ii) local-to-currency pricing (LCP), under which producers selling abroad are assumed to set prices in the currency of consumers rather than their own (Feenstra and Kendall, 1997; Haskel and Wolf, 2001).
2. The original idea dates back to Eli Heckscher (1916) and Gustav Cassel (1922).
3. Recent surveys of foreign exchange intervention include Sarno and Taylor (2001) and Humpage (2003). While there is no conclusive consensus on the overall effectiveness of official intervention, more recent studies (e.g. Fatum and Hutchinson, 2003) demonstrate a high-frequency relationship between foreign market intervention and both the level and change of exchange rates.
4. See also Coakley and Fuertes (2001), who employ a symmetric model to examine market segmentation in Europe.
5. Other nonlinear models exist in the literature. For example, Nicholls and Quinn (1982) discuss random coefficient autoregressive models (RCAR) processes; Granger and Joyeux (1980) introduce fractionally integrated processes, ARFIMA (0,d,0); and Kim (2000) develops a test of whether a process shifts from a stationary to a nonstationary series. We do not consider these models in this paper.
6. Kapetanios and Shin (2002) also propose a direct unit root test designed to have power against globally stationary three-regime self-exciting TAR processes. Their approach differs from that of Caner and Hansen who apply the threshold nonlinearity explicitly to all parameters and use the difference of the series as the transition variable. Neither model explicitly allows for a time-varying threshold.
7. Dahl and González-Rivera (2003) propose new tests that are free of unidentified nuisance parameters under the null of linearity, robust to the specification of the variance–covariance function of the random field, and appear to have superior performance in detecting bilinear, neural network and smooth transition autoregressive specifications.
8. We use the same countries in Dutta and Leon (2002), except for South Africa, which did not satisfy this geographical breakdown.
9. The Caner and Hansen design does not allow for time-varying thresholds. We are unaware of a general asymptotic theory for time-varying thresholds; however, our use of the bootstrap lessens the dependence on an asymptotic theory.
10. Sarno *et al.* (2002), using a similar approach, caution that there may be a cost to over-fitting a TAR model, because the power of Hansen's linearity test was found to be higher the lower the lag length of the TAR.
11. If the true process is stationary, the bootstrap distribution converges in probability to the correct asymptotic distribution. For unit root cases, the asymptotic distribution is discontinuous in the parameters at the boundary where $\rho = 0$ and is not consistent for the correct sampling distribution. Thus, the constrained bootstrap, which ensures that the bootstrap distribution will not be inconsistent for the correct sampling distribution, is first-order asymptotically correct under the null if the true process is a unit root, but incorrect if the true process is stationary.
12. The logistic smooth transition regression (LSTR) is $F(z_t^d;\gamma,c) = [(1+\exp\{-\gamma(z_t^d - c)\})^{-1}]$ and the exponential smooth transition regression (ESTR) is $F(z_t^d;\gamma,c) = [1-\exp\{-\gamma(z_t^d - c)^2\}]$.
13. The more general model $\Delta y_t = \phi' x_{t-1} + \theta' x_{t-1} F_t(z_t^d) + \delta' x_{t-1} F_t(T) + \pi' x_{t-1} F(z_t^d) F_t(T) + u_t$ can be interpreted as describing Δy_t by a STAR model at all times but with a smooth change in the autoregressive parameters from ϕ and θ to δ and π in the regimes corresponding with $F_t = 0$ and $F_t = 1$ (Lundbergh *et al.*, 2003). Allowing for asymmetric speeds of transition between the outer and the middle regimes generates the time-varying BSTR model.
14. These models have been widely used since Hamilton's (1989) application of Markov-switching models to characterize fluctuations in the growth rate of US GDP.
15. Each iteration of the EM algorithm has two steps: (1) the expectation step estimates the unobserved states by their smoothed probabilities; and (2) the maximization step generates estimates of the parameter vector using the smoothed probabilities from the expectation step.
16. We use real effective exchange rates to focus on competitiveness and to avoid issues relating to the choice of numeraire currency (see O'Connell, 1998; Coakley and Fuertes, 2000). Further, because the real effective exchange rate is a weighted average of real

bilateral exchange rates and averaging is more likely to generate stationarity, our results can be interpreted as conservative with respect to a finding of nonstationarity.

17. Following the IMF's World Economic Outlook (WEO) classification, the advanced countries are **G7**: Canada, France, Germany, Italy, Japan, United Kingdom, United States and **Other**: Australia, Belgium, Israel, Korea, New Zealand, Spain. The developing countries are **Asia**: India, Indonesia, Malaysia, Philippines, Thailand and **Western Hemisphere** (**WH**): Argentina, Brazil, Chile, Colombia, Costa Rica, Mexico, Paraguay, Uruguay.
18. Nonnormality in financial data has implications for asset pricing, portfolio choice, value at risk and option valuation (see Jondeau and Rockinger, 2003). For example, nonnormality will affect the usefulness of forecasts if normality is assumed in generating these forecasts, and skewness in preferences of investors may affect the extent of portfolio diversification.
19. There is little theoretical guidance on the value of the delay parameter. While $d=1$ is commonly used, a typical suggestion is to minimize the residual variance over $d=\{1,2,\ldots,d_{\max}\}$. While runs with $d=2,3$ were less satisfactory, we also think $d=1$ is more easily interpretable in our modelling context.
20. See Coakley *et al.* (2003) who propose an algorithm with low computational burden but accurate grid search.
21. While the results for the subregions are similar to Leon and Najarian (2002), overall averages differ in some instances, reflecting the influence of the countries that were included in that study but not included here.
22. We also calculated constrained bootstrap Wald statistics for the Lin vs. TAR. These indicated that if the DGP is a simple unit root process and we tested for linearity (stationary) against TAR, then for some countries we would falsely accept the null too frequently.
23. The terms resulting from a second-order expansion do not allow discrimination among the nonlinear alternatives.
24. We also tested for linearity against the TVBSTAR, using $(F_{Lin})H_0^{LIN}:\theta=\delta=\pi=0$ vs. $H_A^{BTVSTAR}$; $(F_1)H_0^{BSTAR}:\theta=\pi=0$ vs. H_A^{BTVAR}; $(F_2)H_0^{BTVAR}:\delta=\pi=0$ vs. H_A^{BSTAR}; and $(F_3)H_0:\pi=0$.

Few countries' data supported the TVBSTAR.

25. The modelling methodology can be found in Teräsvirta (1994, 1998) and Lundbergh *et al.* (2000).
26. When $\gamma_L=\gamma_H=\gamma^*$ the BSTAR transition function closely approximates the second-order LSTAR model, especially for large values of slope parameter (γ).
27. The detailed test results are available on request.
28. Berben and van Dijk (1998) also find evidence of asymmetric adjustment and conclude that goods arbitrage alone cannot account for nonlinearity in the data.
29. Trade openness is defined as the ratio of exports plus imports to GDP.
30. Lane and Milesi-Ferretti (2001) find evidence that the net foreign position is related to openness, size and level of development, and Granato *et al.* (2002) suggest the existence of more aggressive monetary policy rules in smaller and more open economies.

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